Further Spectroscopic Studies of the Blue component of Albireo

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Abstract

In a previous study of the blue component of the Albireo double star system (AlbireoB) the spectra at H_α and H_β were split into two components, one absorption component from the star itself and a second emission component from the equatorial disk that is being spun off from this rapidly rotating star. To split these spectral components, the star's photosphere was modelled as a single layer in thermal equilibrium at a temperature of 18025K and with a mean free path between particle collisions of 42A. Parameters of the star's rotation were also determined but these parameters do not directly concern us in this study. The aim of this further work was to deduce additional properties of the star's photosphere in particular its pressure, mass density and effective thickness.

1. Introduction

The blue component of the Albireo (β Cyg) double star system (Albireo B), see figure 1, is classed as a B8Ve star i.e. a hot (B8) main sequence (V) star that displays evidence of emission lines (e) in its spectrum.

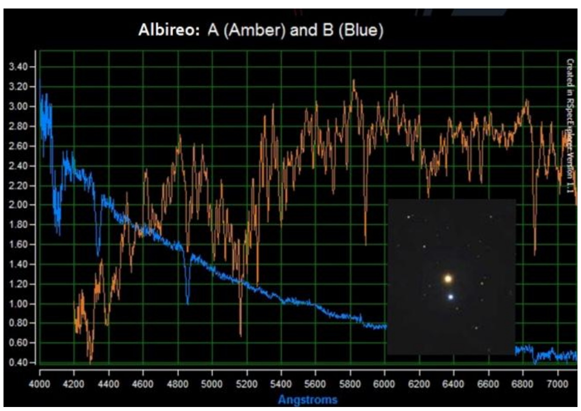

Figure 1: Spectra from the Albireo double star plus, inset, an image of the pair.

A "normal" star would be expected to display only absorption lines, superimposed on the top of the thermal continuum. This absorption is due to continuum photons, at characteristic wavelengths, being absorbed by atoms and then re-emitted in random directions and "thermalised" (returned to the continuum) as they pass through the star's photosphere on their journey to the spectrograph.

The emission component from Albireo B is believed to originate from an equatorial decretion disk spun off from the rapidly rotating star and this emission is then superimposed on top of the normal absorption lines from the star itself.

In a previous work these two spectral line components were split, using a simple physical stellar model, and properties of the star and disk deduced.

The stellar model used was that of a, solid body, rotating uniformly emitting oblate spheroid with a photosphere that is a single layer in thermal equilibrium. It is also assumed that the observed absorption lines are formed solely within this photosphere.

Using this model an effective "black body" temperature can be deduced from low resolution (150 lines/mm) spectra provided proper calibration is performed to correct the continuum spectrum for instrument response and atmospheric absorption. High resolution (2400 lines/mm) investigations of individual line shapes can then be used to determine other model parameters for example, a "mean free path" between particle collisions in the photosphere and the star's speed of rotation.

The key to separating absorption and emission line components for Albireo B was the fact that there was no evidence of emission at the H_γ wavelength and therefore this line could be modelled to establish the physical properties of the star itself. The expected absorption at H_α and H_β wavelengths could then be computed and the pure disk emission lines deduced by dividing measured H_α and H_β lines by the expected absorption line profiles.

A suite of custom computer programs, freely available from the author, was written to perform the required computations which has now been enhance by an additional piece of software that determines more properties of a star's photosphere. Those properties include:-

- Pressure
- Mass density
- Equivalent thickness
- Degree of ionisation.

The theory behind this additional functionality will be described here along with the results obtained for Albireo B.

2. Theory

In this section we will first determine the density and pressure of a target star's photosphere and then go on the determine the its effective width.

2.1 Photosphere density and pressure

We know that the concentration of atoms is the reciprocal of the average volume swept out by an atom between collisions i.e:-

$$N = \frac{1}{\pi r_x^2 l} \quad (2.1)$$

where N is the total number density of neutral and ionised atoms in a photosphere, l is the mean free path (determined from modelling a spectral line) and the "effective" collision cross sectional area $= \pi r_x^2$. Now by splitting the collision cross-section into a neutral and ionized atom part we can write:-

$$r_x^2 = \frac{N_I r_n^2 + N_{II} r_i^2}{N} \quad (2.2)$$

where r_n is the "effective" radius of a neutral atom, r_i is the "effective" radius of an ionized atom, N_I and N_{II} are respectively the neutral and ionised hydrogen concentrations. We can calculate a weighted average classical radius for a neutral atom at any given temperature but, as quantum mechanically an electron wave function has a significant value a considerable distance beyond the classical radius a multiplier (q_m) has been introduced i.e:-

$$N r_x^2 = N_I (q_m r_n)^2 + N_{II} r_i^2 \quad (2.3)$$

Substituting from (2.1):-

$$\frac{1}{\pi l} = N_I (q_m r_n)^2 + N_{II} r_i^2 \quad (2.4)$$

Now the Saha equation states for Hydrogen that:-

$$N_{II}^2 = \frac{N_I}{\Lambda^3} \exp\left(-\frac{E_{ion}}{kT}\right) \quad (2.5)$$

where E_{ion} is the ionisation energy of Hydrogen (13.6eV) and Λ is the electron thermal de Broglie wavelength $\left(\Lambda = \sqrt{\frac{h^2}{2\pi m_e kT}}\right)$.

Substituting from (2.5) into (2.4) we obtain:-

$$\frac{1}{\pi l} = N_{II}^2 \Lambda^3 \exp\left(\frac{E_{ion}}{kT}\right) (q_m r_n)^2 + N_{II} r_i^2 \quad (2.6)$$

So if I know r_i we can solve (2.6) to determine N_{II} and hence N_I , N and the mass density.

For the Sun we know both l (from measurement = 467A) and N (from published data = 1.22e23) so we should be able to determine r_i. To do this re-write (2.4) as:-

$$\frac{1}{\pi l} = (N - N_{II}) N_I (q_m r_n)^2 + N_{II} r_i^2 \quad (2.7)$$

So:-

$$r_i^2 = \frac{\frac{1}{\pi l} - (N - N_{II}) N_I (q_m r_n)^2}{N_{II}} \quad (2.8)$$

Now from (2.5) I know:-

$$N_{II}^2 = \frac{(N - N_{II})}{\Lambda^3} \exp\left(-\frac{E_{ion}}{kT}\right) \quad (2.9)$$

From (2.9), a quadratic in N_{II}, I can determine N_{II} and then use (2.8) to obtain r_i. QED

We can now go on to determine the photospheric pressure, the perfect gas law states that:-

$$PV = nRT \quad (2.10)$$

where P is pressure, V is volume, T is absolute temperature, n is the number of moles of the particles, R (= 8.31441) is the molar gas constant therefore:-

$$P = \frac{n}{V} RT \equiv n_v RT \quad (2.11)$$

where n_v is the number of moles of the particles per unit volume, but N is the number of particles per unit volume (that we now know) so:-

$$P = \frac{N}{N_A} RT \quad (2.12)$$

where N_A is Avogadro's number (= 6.022045e23). An alternative way of writing the same equation is:-

$$P = NkT \quad (2.13)$$

Where k is Boltzmann's constant (=1.380662e-23). Which for the Sun yields:-

$$P_{sun} = 97.2 mBar \quad (2.14)$$

This result for the sun is of the correct order.

2.2 Photosphere thickness

To determine the effective thickness of a photosphere we need to use an additional equation which was derived in the previous study of AlbireoB i.e:-

$$CN_{I2}t = \frac{-\ln[A(\lambda_0)]}{P(T,\lambda_0)d\lambda_0 exp\left[-\left(\frac{hc}{kT\lambda_0}\right)\right]} \quad (2.15)$$

where:-

- C is a constant of proportionality.
- N_{I2} is the number of neutral Hydrogen atoms in the n=2 state
- t is the effective thickness of the photosphere.
- λ_0 is the central wavelength of the spectral line under study.
- $A(\lambda_0)$ is the spectral intensity at the wavelength λ_0.
- $P(T,\lambda_0)$ is the Planck function
- $d\lambda_0$ is a small wavelength range at λ_0 (effectively the CCD bin width)

All other symbols take their usual physical meanings.

Note that we can write:-

$$N_{I2} \approx N_I exp\left[-\left(\frac{hc}{kT\lambda_{12}}\right)\right] \quad (2.16)$$

where λ_{12} is the Lyman alpha wavelength (1216A). This step is justified because most neutral atoms are in the n=1 state even at the higher stellar temperatures for example for AlbireoB with T = 18025K the factor $exp\left[-\left(\frac{hc}{kT\lambda_{12}}\right)\right] = 0.0014$. Equation (2.15) can thus be re-written as:-

$$CN_I t = \frac{-\ln[A(\lambda_0)]}{P(T,\lambda_0)d\lambda exp\left[-\left(\frac{hc}{kT}\left[\frac{1}{\lambda_0}+\frac{1}{\lambda_{12}}\right]\right)\right]} \quad (2.17)$$

When applied to the Sun at H_α (2.17) becomes:-

$$CN_{Isun}t_{sun} = \frac{-\ln[A(\lambda_\alpha)]}{P(T_{sun},\lambda_\alpha)d\lambda_\alpha exp\left[-\left(\frac{hc}{kT_{sun}}\left[\frac{1}{\lambda_\alpha}+\frac{1}{\lambda_{12}}\right]\right)\right]} \quad (2.18)$$

When applied to a target star we have:-

$$CN_{Istar}t_{star} = \frac{-\ln[A(\lambda_0)]}{P(T_{star},\lambda_0)d\lambda_0 exp\left[-\left(\frac{hc}{kT_{star}}\left[\frac{1}{\lambda_0}+\frac{1}{\lambda_{12}}\right]\right)\right]} \quad (2.19)$$

Equations (2.18) and (2.19) can be divided to eliminate the constant of proportionality C and in doing so relate the star's photospheric thickness to the known thickness of the Sun's photosphere ($t_{sun} = 4.0e5m$).

3. Measurements

A high resolution (2400 line/mm) spectrum of AbireoB taken at the H_γ wavelength is shown in figure 2.

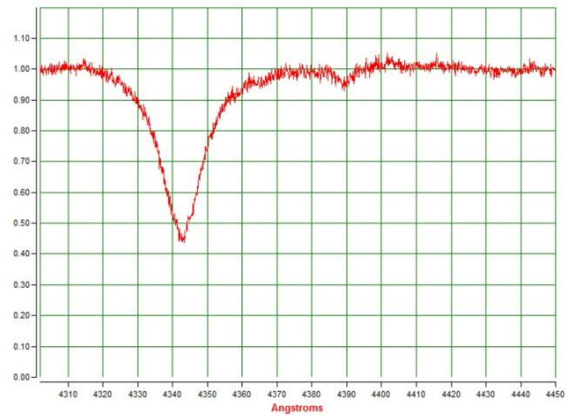

Figure 2: Measured AlbireoB H_γ absorption line

For this H_γ absorption line, using the custom software, the continuum gradient was removed and the central wavelength determined, based on equal areas each side of centre. The $A_\gamma(\lambda_\gamma)$ value for the normalized absorption line was found to be 0.439. The profile was then converted to an equivalent normalized emission line prior to modelling, the result of which is shown in figure 2 in the form of a modelled absorption line.

The theory behind the modelling is not presented here but is available on the RSpec user group website or directly from the author for those interested in the detail. Parameters of the model are:-

- L0: the central wavelength in Angstrom
- T: the photosphere temperature in Kelvin
- MFP: the particle mean free path in Angstrom
- v: the maximum surface velocity of the star as a proportion of c.
- Ob: the Oblateness of the star i.e. equatorial radius divided by polar radius (Ob ≥ 1).
- Theta: the viewing angle in radians above the stellar equator.
- dL: this is the $d\lambda$ value at the central wavelength.
- AMass: I assume the effective atomic mass of the atoms is 1.255 (91.5% H and 8.5% He

atoms by number) a different figure can be input to the simulations if preferred.

All parameter values appear as labels in the RSpec displayed spectra as can be seen in Figure 3, the equivalent width of the absorption is also calculated and displayed as a label.

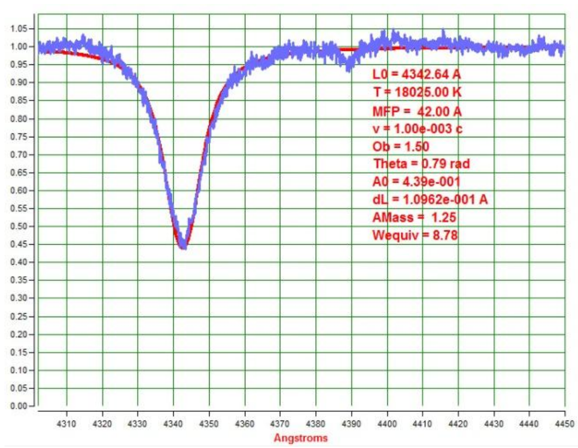

Figure 3: Simulated H_γ absorption line (red) target (blue)

The theory described in section 2 has been implemented in a new piece of software (PhoSph.exe) which when run on the RSpec file displayed in figure 3 results in additional stellar parameters being calculated and output as labels, see figure 4.

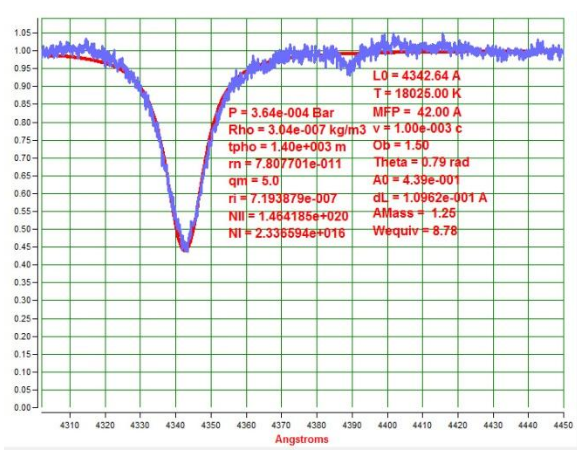

Figure 4: Simulated H_γ absorption line (red) target (blue)

These extra labels display the following properties of the photosphere:-

- *P*: The pressure in Bars.
- *Rho*: the mass density.
- *tpho*: the effective thickness
- *rn*: the weighted average classical atomic radius.
- *qm*: the classical radius multiplier to allow for quantum effects (a runtime adjustable parameter).
- *ri*: the deduced ionised effective atom radius.
- N_{II}: the ionised atom concentration
- N_I: the neutral atom concentration.

4. Discussion

When the new software is run on spectra taken from the sun, via reflection from Jupiter's moon Europa, the results obtained at H_α are displayed in figure 5.

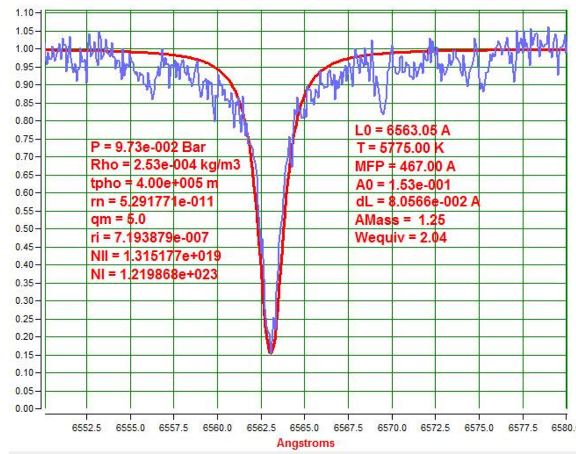

Figure 5: Sun: Simulated H_α absorption line (red) target (blue)

It can be seen that the known properties of the Sun's photosphere are, not surprisingly, accurately reproduced i.e:-

- Mass density = 2.53e-4 kg m^{-3}
- Effective thickness = 4.00e5 m

In order to obtain these values, given the model used with a times five classical atom radius multiplier, the effective ion radius is calculated to be 7.19e-7 m. It is assumed that this value is temperature independent. The calculated pressure is close to the accepted mean value which is of the order of 0.05Bar. It can also be seen that the neutral atoms greatly outnumber the ionised atoms.

Looking now at the results for AlbireoB we see that the pressure is much lower than for the Sun i.e. 0.36mBar. The density at 3.04e-07 kg m^{-3} and thickness at 1.4e3 m are also significantly less than for the Sun. As expected, given the temperature of 18025K, the number of ionised atoms greatly exceed the number of neutral atoms. These numbers suggest that the surface of AlbireoB would look much sharper than the Sun's with less limb darkening.

5. Conclusions

A theoretical analysis of stellar spectra has been presented, based on a simple thermal equilibrium photosphere model, that determines "ball park" figures for parameters of a star's photosphere those parameters being:-

- Pressure
- Density
- Thickness
- State of ionisation

When applied to the star AlbireoB the results obtained are:-

- Pressure = 3.64e-4 Bar
- Density = 3.04e-7 $kg m^{-3}$
- Thickness = 1.4e3 m
- State of ionisation: 98.4% ionised

The model needs to be tested on more stars in order to test its applicability and self consistency. I would invite constructive criticism (via the RSpec usergroup) so that I can correct any errors in the analysis that may exist.

6. Acknowledgements

I would like to acknowledge the many useful email discussions with, fellow RSpec user group member, Mike Bushman particularly concerning Lorents distributions and the Saha equation.

All spectra presented in this paper have been processed and displayed using RSpec software available at www.rspec-astro.com.