Direct Measurement of Acceleration and Higher Order Dynamic Variables from GPS Signals

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Abstract

Relativity Physics teaches us that all physical systems respond in an equivalent way to the effects of gravity and acceleration. Therefore, as acceleration can be measured directly by mechanical means i.e. a mass on a spring, there must exist a direct electromagnetic analogue. This paper will describe the structure and design of just such an analogue and show that it can be usefully realised within the realm of civilian GPS yielding sensitivities of the order of 0.5 $m\ s^{-2}$ (~ 0.05g).

The electromagnetic measurement of acceleration relative to a GPS satellite is achieved by targeting that satellite with two carrier tracking loops. The first loop tracks using a third order Phase Lock Loop (PLL) that is insensitive to acceleration whilst the second tracks with a first order Frequency Lock Loop (FLL) that lags in frequency under constant acceleration conditions. Acceleration can then be deduced from the frequency difference between the two loops. The resulting theory has been validated by system simulations using the SPW[1] analysis tool.

Direct measurement of acceleration can be used to increase the accuracy of position and velocity estimation as obtained for example by Kalman filtering. Further as modern GPS systems comprise 12 or more tracking loop channels, for speed of acquisition, then the channels made redundant following the acquisition phase can be switched to accelerometer mode. Thus the advantages of accelerometer measurements can be obtained for no hardware cost and without the need for additional mechanical systems.

The theory can be extended to the measurement of jerk via a “Jerkometer” and beyond to higher order dynamic variables. However the sensitivity available within the current civilian GPS is too low for useful measurements to emerge in these cases.

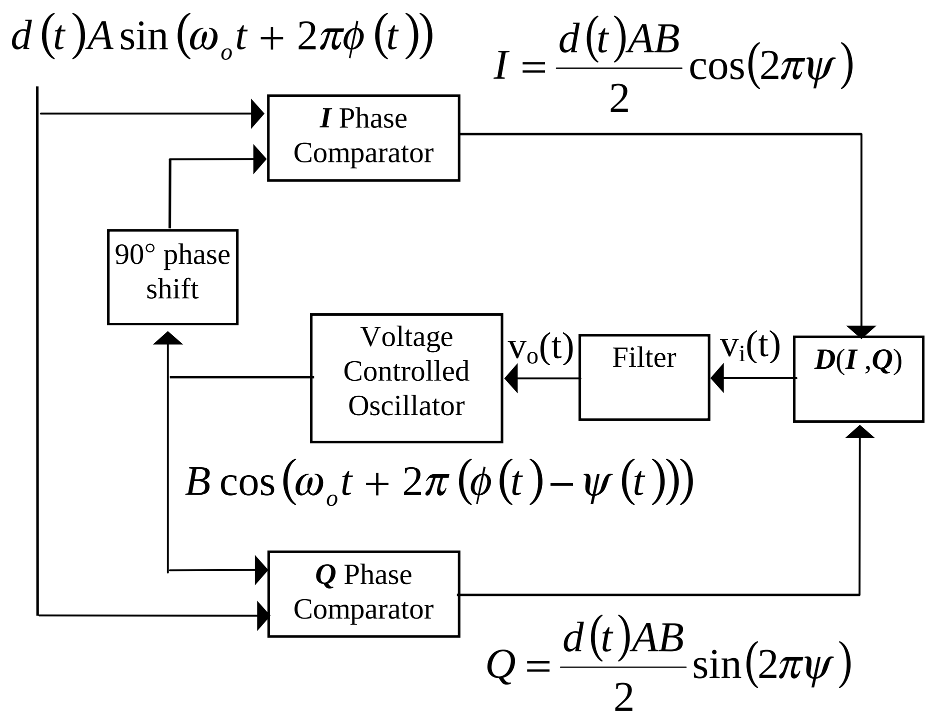
1.0 Introduction

Conventional GPS outputs of position and velocity are rarely presented to the user directly. Most systems employ a filter e.g. Kalman filter, to smooth and refine the output to minimise the effects of noise. Typically these filters work by gathering statistics on past measurements of dynamic variables from which a projection into the future can be made with a known degree of confidence. This projection is combined with a measurement of the dynamic variables at that future time to yield a best estimate of the state of motion of the user as then current. Obviously the more dynamic variables that can be measured the better will be the projection of the current state into the future. For this reason data derived from mechanical accelerometers is often included within a navigational solution, however such accelerometers are relatively bulky and expensive. In this paper an electromagnetic alternative will be proposed to extract acceleration and higher order parameters from the received GPS signal itself. These higher order variable measurements yield independent results and are achieved by targeting individual satellite transmissions with multiple tracking loops of different type and order. Monitoring tracking frequency differences yields the required information.

The requirement for additional tracking loops does not necessarily imply a hardware cost as many GPS chips now on the market carry significantly more tracking loops than are required to track the maximum number of satellites that are visible at any one time, a number typically between eight and ten. The excess loops are used to speed initial acquisition at system startup and, being generally implemented in the digital domain, can be reprogrammed to function as higher order motion sensors in later phases of system operation. Any additional hardware cost would in any case be insignificant compared to the mechanical alternative.

To understand the current proposal we need to understand the process by which a navigational satellite transmission is tracked. Due to the BPSK data encoding method employed in the GPS system Costas tracking loops are generally used, spread spectrum techniques are also employed but we shall not explicitly include code synchronisation in our discussion as it does not alter the basic frequency tracking process. Analogue carrier tracking via a Costas loop is illustrated in figure 1.

Figure 1: Costas Carrier Tracking Loop



An incoming signal at angular frequency ω_0 exhibits phase modulation as a function of time represented by $\phi(t)$, the signal also exhibits essentially random 180° phase shifts due to the BPSK data modulation represented by the parameter $d(t) = \pm 1$. In phase $\boldsymbol{I}$ and quadrature phase $\boldsymbol{Q}$ signals are generated in a phase comparator by multiplication with cos and sin signals, generated by a voltage controlled oscillator, followed by high frequency filtering to remove components at approximately $2\omega_0$. A discriminator function $D(I,Q)$ produces an output voltage $\mathrm{v_i}$ that gives a measure of ψ, typically $D(I,Q) = Arc\tan\left(\frac{Q}{I}\right) = \psi$.

These measured phase errors are filtered to produce an output voltage $\mathrm{v_o}$ that controls the oscillator. In this way the loop is closed and the response of the loop depends on the filter function.

This paper will describe how the form of the filter function affects the way in which the loop tracks, it will also demonstrate how an accelerometer can be realised by combining the output of two particular types of loop. Practical implementation will then briefly be considered before presenting system simulation results to confirm the theory developed here. Extension to higher order dynamic variable measurement will then be described.

2.0 Theory

The differential equation that governs the response of a Costas type linear i.e. $D(I,Q) = \psi$ carrier tracking loop is[2]:-

$$\frac{d\psi}{dt} + F(\psi) = v(t) \tag{1}$$

where ψ is the loop phase error in circles, $v(t)$ is the time dependent loop forcing frequency in Hz (equal to $\frac{d\phi}{dt}$ see figure 1) and $F(\psi)$ is the loop filter function. The exact form of $F(\psi)$ determines the type of loop i.e. PLL or FLL as well as its order. A PLL aims to track signal phase and therefore also frequency whilst an FLL aims only to tracks the signal frequency. Given the type of loop its order determines how it responds to dynamic stress i.e. whether it tracks the received signal accurately, with a constant offset or loses lock altogether after a period of time.

A PLL results when the filter function in equation (1) takes the form:-

$$F(\psi) = c_{n-1}\psi + c_{n-2}\int \psi\, dt + \Lambda + c_0 \int \Lambda \int \psi\, dt^{n-1} \tag{2}$$

Whilst an FLL results when the filter function in equation (1) takes the form:-

$$F(\psi) = C_{n-1} T \frac{d\psi}{dt} + C_{n-2} T\psi + \Lambda\ + C_0 T \int \Lambda\ \int \psi \, dt^{n-2} \tag{3}$$

where the filter coefficients c_i, C_i and T are constants. The reason for factoring out a constant T (of units seconds) in the coefficients of equation (3) is that the coefficients c_i and C_i then have the same units for a given value of i i.e. seconds to the power $i - n$. In practice the value of T will be associated with the system time step between measurements of phase error as then

$$T \frac{d\psi}{dt} \approx \psi_i - \psi_{i-1} \equiv \beta_i \tag{4}$$

where the ψ_i are discrete time system measurements of phase error.

2.1 Critically Damped PLL's

To solve equation (2) given the nth order filter function (2), we differentiate $n-1$ times to obtain:-

$$\frac{d^n \psi}{dt^n} + c_{n-1} \frac{d^{n-1}\psi}{dt^{n-1}} + \Lambda\ + c_0 \psi = \frac{d^{n-1} v}{dt^{n-1}} \tag{5}$$

Solutions to equation (5) are constructed from solutions to the corresponding homogeneous equation (where the right hand side is zero). To solve the homogeneous form of (5) we make the substitution: -

$$\psi = \psi_0 e^{mt} \tag{6}$$

From which we obtain: -

$$m^n + c_{n-1} m^{n-1} + \Lambda\ + c_0 = 0 \tag{7}$$

For an nth order PLL there will in general be n distinct roots of equation (7). Note however that as the coefficients c_i are real then the roots of (7) must be real or occur in complex conjugate pairs. In a critically damped loop the n roots of (7) are further restricted to the values $-a, -a(1+i)$ and $-a(1-i)$ where a represents the inverse of the critically damped loop time constant.

In this way we may determine the filter coefficients, in terms of the loop constant a, for critically damped loops of arbitrary order with the result given in table 1 for orders up to 3. In this table we have chosen to use the complex conjugate root pairs where possible as this maximises the coefficient c_0 and therefore minimises the steady state phase error offsets under dynamic forcing as can be deduced from equation (5).

Table 1: Critically Damped Loop Coefficients for Orders up to 3

Loop Order	c_2	c_1	c_0
1	-	-	a
2	-	$2a$	$2a^2$
3	$3a$	$4a^2$	$2a^3$

2.2 Critically Damped FLL's

Substitution of (3) into (1) and differentiation $n-2$ times yields:-

$$\frac{d^{n-1}\psi}{dt^{n-1}}+\frac{C_{n-2}T}{1+C_{n-1}T}\frac{d^{n-2}\psi}{dt^{n-2}}+\Lambda+\frac{C_0T}{1+C_{n-1}T}\psi=\frac{1}{1+C_{n-1}T}\frac{d^{n-2}v}{dt^{n-2}} \qquad (8)$$

and it is the coefficients of equation (8) that need to be chosen appropriately, according to Table 1, to yield critical damping. Note that given a value for n, the order of an FLL is here defined to be equal to $n-1$ as $n=2$ is the minimum value required to yield a meaningful FLL equation.

2.3 Accelerometer

An accelerometer can be constructed from a critically damped PLL of at least 3rd order i.e. with $n=3$ in equation (5), and a 1st order FLL with $n=2$ in equation (8).

Under constant acceleration the received frequency varies linearly with time and assuming the acceleration is initiated at time $t=0$, the forcing term takes the form:-

$$v(t)=\frac{f\,A}{c}t\equiv v_1 t \qquad (9)$$

where A is the line of sight acceleration of the receiver relative to the transmitter, f is the frequency of transmission in the transmitter's rest frame and c is the speed of light.

The resulting governing equations are therefore for the 3rd order PLL:-

$$\frac{d^3\psi}{dt^3}+3a\frac{d^2\psi}{dt^2}+4a^2\frac{d\psi}{dt}+2a^3\psi=0 \qquad (10)$$

and for the 1st order FLL:-

$$\frac{d\psi}{dt}+\frac{2b^2T}{1+2bT}\psi=\frac{v_1 t}{1+2bT} \tag{11}$$

where b is a positive constant. We have chosen to set $C_0 = 2b^2$ and $C_1 = 2b$ so that the 1st order FLL may be realised simply by substituting the measured discrete time relative phase errors $\psi_i - \psi_{i-1} \approx T\frac{d\psi}{dt}$ for the absolute phase errors ψ_i in the filter function of a critically damped 2nd order PLL.

In practice we choose a value for the constant a according to the expected system dynamics and noise environment. To determine a value for b note that we need to match the time constants of the two loops therefore we require:-

$$a=\frac{2b^2T}{1+2bT} \tag{12}$$

from which we deduce:-

$$b=\frac{a+\sqrt{a\left(a+\frac{2}{T}\right)}}{2} \tag{13}$$

equations (10) and (11) can be solved to yield the following phase errors as a function of time:-

$$\psi=\frac{v_1 e^{-at}}{a^2}\{1-\cos(at)\} \tag{14}$$

for the PLL and:-

$$\psi=\frac{v_1}{2b^2T}\left\{t-\frac{\left(1-e^{-at}\right)}{a}\right\} \tag{15}$$

for the FLL. Differentiating equations (14) and (15) with respect to time we find that:-

$$f_{error}(t)=\frac{d\psi}{dt}=\frac{v_1 e^{-at}}{a}\{\cos(at)+\sin(at)-1\} \tag{16}$$

for the PLL and:-

$$f_{error}(t) = \frac{d\psi}{dt} = \frac{v_1}{2b^2T}\left(1 - e^{-at}\right) \tag{17}$$

for the FLL. As $t \to \infty$ we see that the PLL tracks with a zero frequency error whilst the FLL tracks with a constant frequency error given by:-

$$\Delta f \equiv f_{error}(\infty) = \frac{v_1}{2b^2T} \tag{18}$$

Hence, using equation (9) we may deduce that:-

$$A = \frac{2b^2Tc}{f}\Delta f \tag{19}$$

Therefore the receiver to transmitter line of sight acceleration can be obtained from the frequency difference between the PLL and FLL loops. Such measurements to four or more GPS satellites can be used to determine the user's spatial acceleration and clock acceleration.

Figures 2 and 3 plot the phase and frequency errors respectively computed from equations (14) to (17) given a line of sight acceleration of 15 ms^{-2} (~1.5g) and loop parameters as stated in the figures.

Figure 2: Tracking Loop Phase Error vs Time

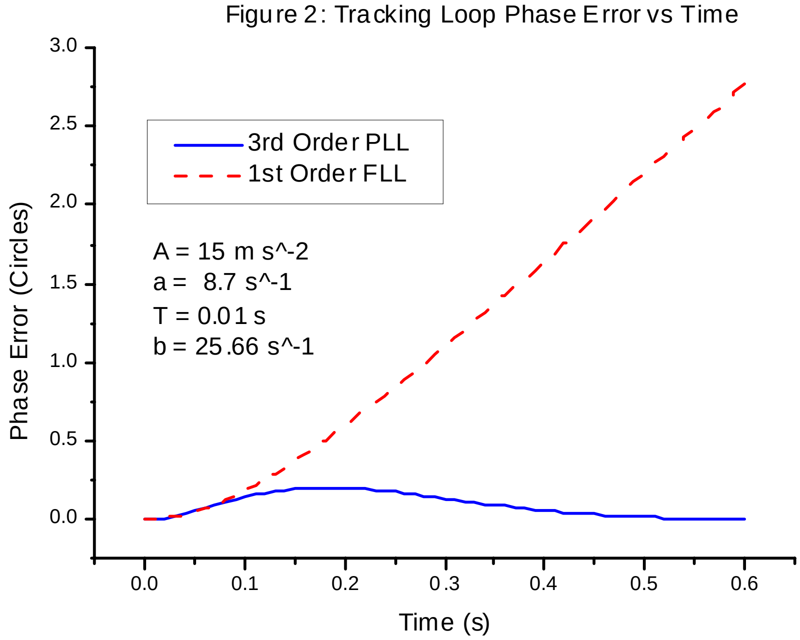
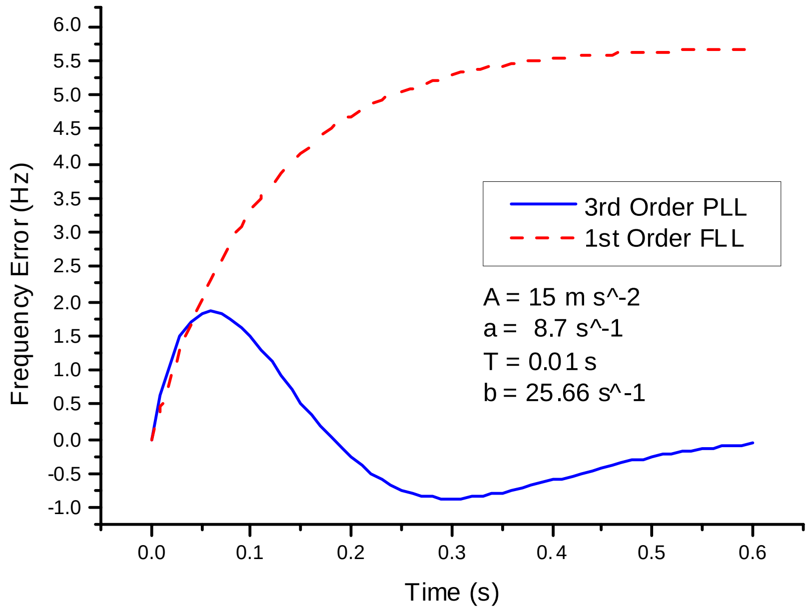


Figure 3: Tracking Loop Frequency Error vs Time



3.0 Accelerometer Implementation

The loops described so far have been continuous in time (i.e. analogue) however in practice tracking loops are best implemented in the digital domain following A to D conversion and sampling of the output from an RF front-end. Such digital loops have the advantage that they can be easily re-configured. Loop implementation therefore starts from a discretisation of the loop equations (1), (2) and (3) which become respectively:-

$$\frac{\psi_i - \psi_{i-1}}{T} + F(\psi_i) = v(iT) \tag{20}$$

for the governing equation,

$$F(\psi_i) = 3a\psi_i + 4a^2T\left\{\sum_{j=0}^{i-1}\psi_j + \frac{\psi_i}{2}\right\} + 2a^3T^2\left\{\sum_{j=0}^{i-1}\sum_{k=0}^{j}\psi_k + \frac{\psi_i}{6}\right\} \tag{21}$$

for the 3rd order PLL filter function and, following substitution of relative phase errors into the filter function of a 2nd order PLL, the 1st order FLL filter function becomes:-

$$F(\beta_i) = 2b\beta_i + 2b^2T\left\{\sum_{j=0}^{i-1}\beta_j + \frac{\beta_i}{2}\right\} \tag{22}$$

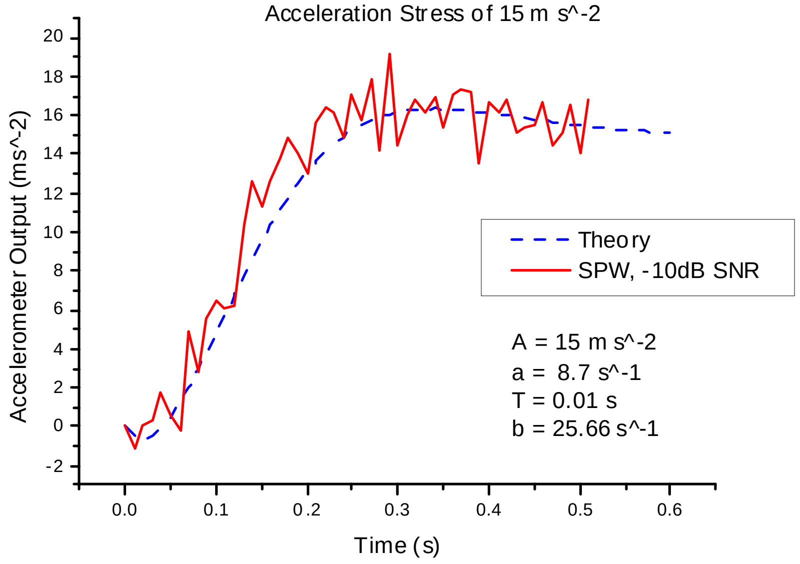
The filter functions (20) and (21) then represent the frequencies required from two NCO's (numerically controlled oscillators) that produce the feed back inputs to digital phase comparators thus following a similar process to that depicted in figure 1.

4.0 Accelerometer Simulation

A complete GPS baseband processing SPW system model was modified to demonstrate the capabilities of the proposed accelerometer.

Figure 4 compares the SPW modelled output of the accelerometer with that predicted from theory. The theoretical curve differences the loop frequencies depicted in figure 3 and scales the result according to equation (19). The SPW output includes the effects of noise at an SNR of –10dB, this level of noise is if anything rather large as Doppler shifts of the order of ±0.1 Hz can normally be resolved on a real system. We see from this figure that the two curves agree and given that a 1.5g acceleration produces an approximate 6Hz frequency difference (figure 3), together with a practical frequency resolution of ±0.1 Hz, then it should be possible to resolve accelerations to 0.05g.

Figure 4: Accelerometer Output vs Time Under an Acceleration Stress of 15 m s^-2



5.0 Higher Order Motion Sensors

It is possible to construct sensors for higher order motion parameters e.g. jerk (rate of change of acceleration). If we define a velocity sensor as 1st order and an accelerometer as 2nd order then an $n-1$th order motion sensor would be constructed from at least an nth order PLL and an $n-2$th order FLL, thus for a "jerkometer" we require $n=4$. In these higher order cases the coefficients of the particular loops would be chosen from an extension of table 1.

For the PLL, odd orders would include a real root, the other roots would be multiple copies of the complex conjugate root pair. In this way transients would be minimised. Whilst for the FLL we would like to maximise the steady state offset of the loop, to improve resolution, and so multiple copies of the real root should be considered. Again we would require that the PLL and FLL loop time constants were matched so the loop coefficients of the $n-1$th order PLL, that implements the $n-2$th order FLL, would need to be adjusted to yield the required critically damped FLL.

As an example Figure 5 shows the tracking performance of a 4th order PLL and two 2nd Order FLL's one with complex roots and one with two copies of the real root (a = 8.7). Immediately we see that whilst the frequency error in the real root case is twice that of the complex root case, both resultant frequency errors are quite small especially given that the 10 m s^{-3} applied jerk which represents quite a high dynamic stress.

Figure 5: Loop Response to 10ms^-3 Jerk

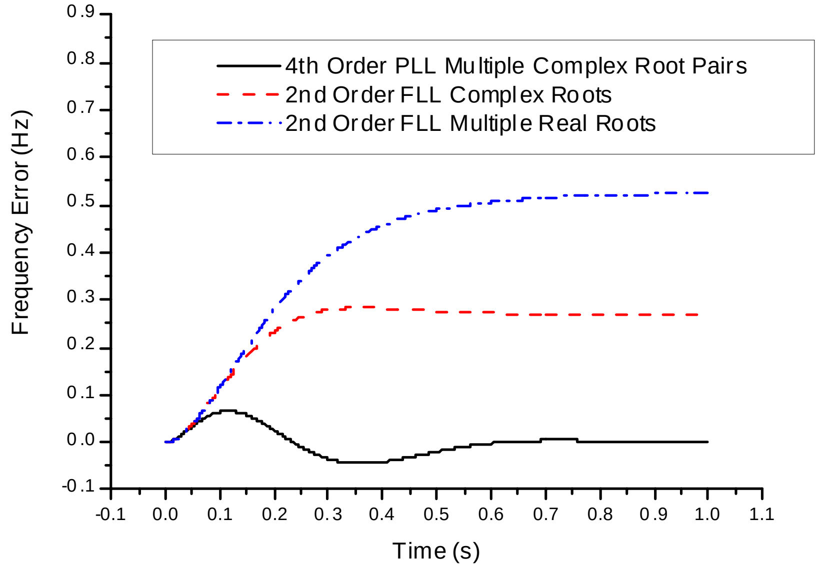
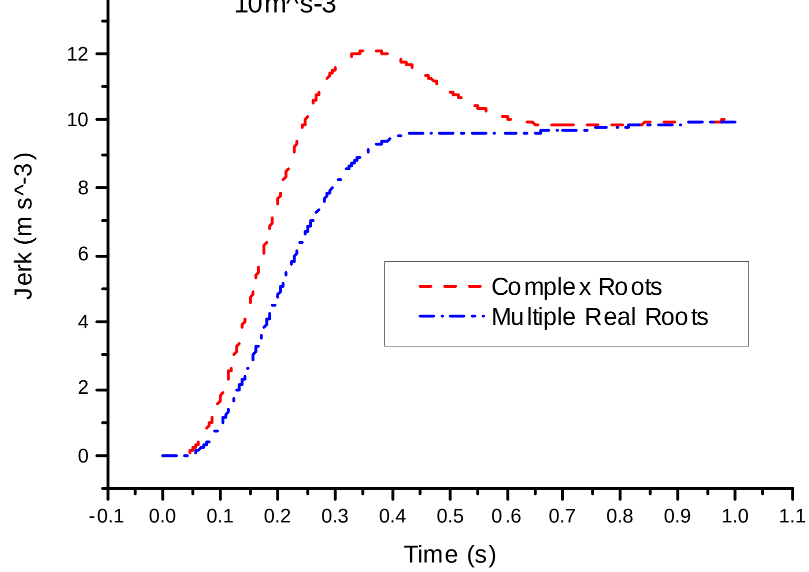


Figure 6: Jerkometer Output at an Applied Jerk of 10m^s-3



6.0 Summary

It has been shown that an accelerometer and higher order motion sensors can be constructed that derive their outputs purely from the received electromagnetic transmissions of navigation satellites. Such outputs are independent measurements and the ability to extract such data is to be expected from the equivalence principle of relativity.

Measurements are derived by targeting multiple tracking loops at individual satellites. A PLL of sufficient order to be insensitive to the dynamic variable of interest is used together with an FLL that exhibits a constant offset in frequency under conditions in which the particular dynamic variable is non-zero. Differencing the PLL and FLL running frequencies then yields a measure of the value of the dynamic variable. The resolution obtained within civilian GPS is adequate for acceleration (~ 0.05g) but may not be adequate for higher order variables depending on the magnitude of expected dynamic stresses within a given application area.

Information on acceleration and higher order parameters can be used to improve the performance of filtering techniques that yield best estimates of the user position and state of motion in the presence of noise. The ability to put such functions on chip saves the considerable cost of mechanical sensors. The cost is further reduced by the fact that many current GPS chipsets, for speed of initial acquisition, already implement more tracking loops than are necessary to track the maximum number of satellites that are visible at any one time. The software nature of these loops also means that they can easily be reprogrammed to perform motion sensor functions as they become free.

References

1. SPW Cadence Design Systems Inc., 555 River Oaks Parkway, San Jose, CA 95134, USA.

2. Principles of Communication Systems second edition, Herbert Taub and Donald Schilling, McGraw-Hill, 1986, ISBN 0-07-062955-2